



# Modeling of Mass Transfer Boundary Layer Instability in the CO<sub>2</sub>-Vapex Process

M. JAVAHERI and J. ABEDI

Department of Chemical and Petroleum Engineering, University of Calgary, 2500 University Drive NW, Calgary, AB Canada, T2N 1N4

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## Abstract

*Vapex (vapor extraction) is a promising technique for the recovery of heavy oil and bitumen reservoirs, especially for cases where steam-assisted gravity drainage and other thermal recovery methods are not economical. In the Vapex process, a solvent is injected into the reservoir to reduce the oil viscosity and mobilize it towards the production well. CO<sub>2</sub>-based Vapex is an attractive option from both economical and environmental perspectives. In CO<sub>2</sub>-based Vapex, unlike other hydrocarbon solvents, the dissolution of CO<sub>2</sub> in oil can result in a density increase of the diluted oil. As a consequence, the diluted oil has a higher density than the immobile oil beneath and a gravitationally unstable diffusive boundary layer is induced, which may lead to natural convection. In this paper, a mathematical model for the diffusive boundary layer in the CO<sub>2</sub>-oil contact region is developed; and, the possibility of convective mixing is examined using linear stability analysis, based on the amplification of the initial perturbations. It is found that in most experimental cases, depending on the Rayleigh number of the porous medium, convective mixing*

*occurs, which results in higher dissolution of CO<sub>2</sub> in oil and thus a higher oil production rate than what is expected from theoretical analysis. This would explain the unexpected higher oil production rate of some experiments in Vapex when CO<sub>2</sub> was used as a solvent. In field-scale operations, the results are different. In field cases, since it is almost impossible for the Rayleigh number to exceed the critical Rayleigh number ( $Ra_c$ ), convection does not happen.*

## Introduction

The world's total reserve of heavy oil and bitumen is about six trillion barrels, which is about six times the amount of the conventional resources<sup>[1]</sup>. A major part of these resources is in Canada, Venezuela and the United States. Most of these reserves are at such depths that open-pit mining cannot be used economically, and in-situ methods have to be used to reduce the viscosity of the oil in-place and mobilize it. Either thermal methods or non-thermal methods can be used to recover these

reserves. The viscosity of oil is a strong function of temperature and decreases sharply with increasing temperature. Currently steam-assisted gravity drainage (SAGD), a thermal method, is a popular method for the recovery of heavy oil and bitumen and has been successfully applied in several fields. Despite the success of this process for some reservoirs, there are many reservoirs that SAGD cannot be applied due to excess heat loss, which makes it uneconomical to operate. In thin reservoirs, the need for steam increases and the steam-to-oil ratio (SOR) is prohibitively high. Many oil and bitumen reservoirs have a bottom aquifer, and heat loss to the water can make the process infeasible<sup>[2]</sup>. There are also reservoir conditions where SAGD may not be applied, such as when water saturation is high, or porosity is low. In cases where SAGD cannot be applied, Vapex is the most promising technique for the recovery of these resources.

The initial development of the Vapex process was first introduced by Butler and Mokrys<sup>[3]</sup>, as a solvent analogue to steam-assisted gravity drainage. The steam chamber in SAGD is replaced by a solvent chamber in Vapex. In the Vapex process, a solvent is injected near its dew point (where both solubility and diffusivity of the vapor solvent into oil are at their maximums) and forms a solvent chamber within the reservoir<sup>[4]</sup>. Dissolution of solvent into oil reduces its viscosity and mobilizes it towards the production well. The rate of solvent diffusion into the oil and the factor of viscosity reduction by solvent are the main parameters that control the production rate in Vapex.

One of the main problems in Vapex is the cost of the solvent, which affects the economics of operation if solvent is lost during the process. An attractive option seems to be the use of CO<sub>2</sub> as one of the main components in Vapex<sup>[5]</sup>. It is beneficial both environmentally and in terms of recovery, since it is more soluble than methane in heavy oil. The saturation pressure of CO<sub>2</sub>, its solubility in heavy oils, its price compared to other solvents, and the environmental effects to reduce greenhouse gas emissions can favor the use of CO<sub>2</sub> in the Vapex process, known as CO<sub>2</sub>-based Vapex<sup>[6]</sup>.

In an experiment done by Dunn et al.<sup>[7]</sup>, they used low-temperature soluble gases to recover bitumen from a glass bead packed cell. In their experiments, they unexpectedly noticed that the drainage rate of the bitumen was higher when CO<sub>2</sub> was used as the solvent than the drainage rate when ethane was used. This was surprising, because the viscosity of bitumen saturated with ethane was about one-fifth of that for CO<sub>2</sub>, and the drainage rate using ethane should theoretically be 2.2 times that of CO<sub>2</sub>. They tried to explain this phenomenon by stating that the dispersion coefficient in a porous medium can be much larger than the molecular diffusivity, which results in a higher drainage rate. This is true and, since the diluted oil is flowing in the mass boundary layer, the dispersion coefficient is much larger than molecular diffusion, but this increase also happens when ethane is used as a solvent. Based on correlations, the dispersion coefficient is a function of the Peclet number in the porous medium, which is proportional to the velocity of the diluted oil draining to the production well. This increase in production rate in the CO<sub>2</sub>-based Vapex process implies that there is another mechanism apart from molecular diffusion and lateral dispersion that results in more CO<sub>2</sub> to be dissolved into the heavy oil or bitumen and, in turn, the achievement of a higher production rate.

Hydrocarbon solvents, such as methane, ethane, propane and butane, can be used to dilute heavy oils and reduce their viscosity. They all decrease the density of heavy oils when dissolved in them. CO<sub>2</sub> can be different from other solvents and

may increase the density of heavy oils. This can happen especially at low temperatures, where the density of the liquid and the solubility of CO<sub>2</sub> into the oil are high<sup>[8]</sup>, and at high pressures, where the density of CO<sub>2</sub> gas is high. Sayegh et al.<sup>[9]</sup> acquired phase behavior and physical properties data of carbon dioxide/Lindberg heavy oil systems. Their phase behavior results show that, when CO<sub>2</sub> dissolves into the oil, the density of the mixture increases with pressure increases and can be higher than the density of the heavy oil. The viscosity of the crude was more than 12 thousands centipoises. Miller and Jones<sup>[10]</sup> measured physical characteristics of three heavy oil samples with and without carbon dioxide. They conducted experiments at three different temperatures – 75, 140, and 200°F. At 75°F, dissolution of CO<sub>2</sub> in Wilmington heavy oil (17 API gravity) increased density at higher pressures, but for high temperatures, the density of the CO<sub>2</sub> /heavy oil mixture was less than the density of the oil. Figure 1 shows the variation of the density of the Wilmington heavy oil and carbon dioxide as a function of pressure, as reported by Miller and Jones. Srivastava et al.<sup>[11]</sup> reported an increase of density of heavy oil when CO<sub>2</sub> was dissolved into it.

The increase of density of oil by CO<sub>2</sub> dissolution may lead to natural convection if the Rayleigh number of the system (which depends on the rock and CO<sub>2</sub>/heavy oil physical properties) is above a critical value. In this case, the mechanism of mass transfer is not diffusion alone, and convection causes more CO<sub>2</sub> to be dissolved from the interface of the diluted oil to the deeper parts of the layer. With convection, the dilution of the oil happens faster, and the production rate increases.

In this paper, we investigate the possibility of natural convection occurrence in the Vapex process when CO<sub>2</sub> is used as a solvent or is the main component in the process. A concentration profile of the solvent is considered in the mass boundary layer of the solvent diffusing into the oil. Linear stability analysis is applied to the governing equations, and the growth of perturbations in the system is calculated. Based on the analysis, the conditions that result in convection are explained.

## Boundary Layer in Vapex

The mechanism of dilution in Vapex occurs in a thin layer of oil that the concentration of the solvent varies. To understand the performance of the Vapex, one has to consider carefully the effect of solvent on heavy oil in this boundary layer. A cross section of this layer is shown in Figure 2 with the coordinate system shown. The coordinate system is a moving coordinate system in which the origin is moving with the interface. The movement of the coordinate system does not have a significant effect on the equations, because the change of the coordinate is small compared to the instability growth in our study, as the results show. This happens because the rate at which the process is operated is not high; and, by the time when the instabilities have grown significantly, only a small change in the angle of the interface has occurred.

In this study, the effect of capillary pressure has not taken into account. The interfaces of solvent chamber/mobile oil and mobile oil/immobile oil are assumed to be flat surfaces. The process is taken to be a pseudo-steady state in which only the driving force for production (gravity force) varies, because the angle of the mobile oil strip with the horizontal is changing. A transient model with surface renewal can lead macroscopically to a steady-state model, as discussed by Das and Butler<sup>[12]</sup>.

The movement of the boundary layer in this study is similar to the LINDRAIN model in SAGD. In order to adjust the flow rate to match experiments in the SAGD process, the coefficient in the production term is reduced. The bottom of the steam chamber boundary in SAGD moves away from the production well, and the oil has to travel a distance horizontally, and this reduces the total head for production. Hence, the coefficient in the production term is reduced in TANDRAIN and LINDRAIN models<sup>[13, 14]</sup>.

In order to account for the variation in the driving force, the change of the slope of the mobile layer with time should be considered. Figure 3 shows the effect of drainage on the slope change. The change of the volume of the drained oil in time can determine the production rate.

At time  $t$ , the angle between the interface and the horizontal is  $\theta$ ; and, at time  $t+dt$ , this angle will be  $(\theta-d\theta)$ . The area swept is:

$$dS = \left( \frac{r \cdot d\theta}{2 \cdot \sin \theta} \right) (H) = \left( \frac{H \cdot d\theta}{2 \cdot \sin^2 \theta} \right) (H) = \frac{H^2 \cdot d\theta}{2 \cdot \sin^2 \theta} \dots\dots\dots (1)$$

where  $H$  is the height of the formation, and  $r$  is the length of the interface strip. The production rate is the rate of the oil and solvent in it. Since here we only examine the change of the angle of the oil/solvent chamber interface, the rate of oil is sought. The oil drainage rate is:

$$q = -\frac{dV}{dt} = -w_1 \cdot \frac{dS}{dt} = -\frac{w_1 \cdot H^2}{2 \cdot \sin^2 \theta} \cdot \frac{d\theta}{dt} \dots\dots\dots (2)$$

If the rate is constant, then:

$$\begin{aligned} \frac{d\theta}{dt} &= -\frac{2q}{w_1 \cdot H^2} \cdot \sin^2 \theta \\ &= -\frac{4\sqrt{1.5kg\phi^\Omega \Delta S_o N_s H}}{H^2} \cdot \sin^2 \theta = -R' \cdot \sin^2 \theta \dots\dots\dots (3) \end{aligned}$$

where  $w_1$  is the length of the horizontal wells,  $R'$  is a constant, and  $q$  is substituted by the appropriate term. In dimensionless form, the above equation will be:

$$\frac{d\theta}{dt_D} = -\frac{4 \cdot L^2 \sqrt{1.5kg\phi^\Omega \Delta S_o N_s H}}{D_1 \cdot H^2} \cdot \sin^2 \theta = -R \cdot \sin^2 \theta \dots\dots\dots (4)$$

where  $t_D = \frac{t D_1}{L^2}$ ,  $D_1$  is the solvent diffusivity in the mobile oil, and  $R$  is a dimensionless constant. For the change of the slope, this dimensionless  $R$  will be used.

## Solvent (CO<sub>2</sub>) Concentration in the Mass Boundary Layer

The rate of the Vapex process is dependent on the rate of mass diffusion of the solvent into the interface of the oil and solvent chamber. As the solvent diffuses into the oil, it lowers the viscosity and mobilizes the oil towards the production well using gravity. For an analysis of the boundary layer in the Vapex, we have to know the concentration profile of the solvent in the boundary layer.

Figure 2 shows the boundary layer of Vapex, where the direction of solvent diffusion into oil and diluted oil movement are perpendicular. At the solvent/heavy oil interface, the concentration of the solvent is at the solubility concentration of the solvent into the oil. With the movement of the boundary layer during production, this interface concentration of solvent

will not vary. We model this layer by a smooth layer with constant thickness through the length of the strip along the interface during operation. During production, the slope of this layer changes. The production of the oil is due to dilution of the oil in the mobile layer. In the immobile layer, the concentration of the oil is not sufficient to force the oil to flow. The diffusion coefficient of the solvent into oil depends on the viscosity of the oil; and, as the viscosity increases, the molecular diffusion coefficient decreases. Hiss and Cussler<sup>[15]</sup> and Hayduk and Cheng<sup>[16]</sup> have proposed relationships between the molecular diffusivity of some solvents and the oil viscosity. Therefore, in the mobile layer, the diffusivity of solvents into diluted oil,  $D_1$ , is more than the diffusivity of solvent into the immobile oil,  $D_2$ . Li et al.<sup>[17]</sup> have proposed a concentration profile for the solvent into the mobile oil layer. If we take the coordinate system as the one shown in Figure 2, the concentration profile can be obtained. If we let  $c_1(z,t)$  and  $c_2(z,t)$  be the concentrations of the solvent in the mobile and immobile oil, respectively, then the following equations can be written for the concentration profile:

$$\frac{\partial c_i}{\partial t} = D_i \frac{\partial^2 c_i}{\partial z^2} \quad i = 1, 2 \dots\dots\dots (5)$$

where  $i = 1$  is for the mobile layer from zero up to the thickness of the mobile layer ( $L$ ), and  $i = 2$  is for the immobile layer from  $L$  to infinity. Equation (5) can be written in dimensionless form as:

$$\frac{\partial c_{Di}}{\partial t_D} = \frac{\partial^2 c_{Di}}{\partial z_D^2} \dots\dots\dots (6)$$

where  $t_D = \frac{t D_1}{L^2}$ ,  $z_D = \frac{z}{L}$ ,  $c_{Di} = \frac{c_i}{c_s}$ , and  $c_s$  is the solvent

solubility in the oil.

The boundary conditions are:

$$c_{D1} = 1, \text{ at } z_D = 0 \dots\dots\dots (7)$$

$$c_{D1} = c_{D2}, \text{ at } z_D = 1 \dots\dots\dots (8)$$

$$D_1 \frac{\partial c_{D1}}{\partial z_D} = D_2 \frac{\partial c_{D2}}{\partial z_D}, \text{ at } z_D = 1 \dots\dots\dots (9)$$

$$c_{D2} = 0 \text{ as } z_D \rightarrow \infty \dots\dots\dots (10)$$

It should be noted that  $z_D = 1$  represents the interface of mobile and immobile oil. At the interface of oil/solvent chamber, the concentration of the solvent is at the solvent solubility concentration,  $c_s$ ; therefore, the dimensionless concentration is 1.

The initial condition is:

$$c_{Di} = 0, \quad i = 1, 2, \quad t = 0 \dots\dots\dots (11)$$

The solutions of Equation (5) with the above boundary and initial conditions are<sup>[18]</sup>:

$$c_{D1} = \sum_{n=0}^{\infty} \alpha^n \left( \operatorname{erfc} \frac{(2n+1) + (z_D - 1)}{2\sqrt{t_D}} - \alpha \operatorname{erfc} \frac{(2n+1) - (z_D - 1)}{2\sqrt{t_D}} \right) \dots\dots\dots (12)$$

$$c_{D2} = \frac{2\kappa}{\kappa+1} \sum_{n=0}^{\infty} \alpha^n \left( \operatorname{erfc} \frac{(2n+1) + \kappa(z_D - 1)}{2\sqrt{t_D}} \right) \dots\dots\dots (13)$$

where  $erfc$  is the complementary error function,  $\kappa = \sqrt{\frac{D_1}{D_2}}$  is

the square root of the ratio of diffusivities of the solvent into mobile and immobile oil, and  $\alpha = \frac{1-\kappa}{1+\kappa}$ .

Equations (12) and (13) are transient equations in which the concentration of the solvent varies with time in both the mobile and immobile layers. During production in Vapex, there is always a mobile layer in which the viscosity of oil is reduced. At one edge of the layer, the concentration is constant; and, at the other edge, there is a specific concentration in which the mobile and immobile oils are separated. These transient profiles are for the start of the analysis; and, when the solvent chamber forms, the concentration profile in the mobile oil does not vary. But in Equations (12) and (13), the transient component,  $t_D$ , exists; and, the solvent in the chamber diffuses to the oil layer, according to these equations. After a specific time,  $t_D^*$ , the solvent concentration profile in the layers does not vary with time in the coordinate system of the problem. This specific time depends on the extent to which the solvent reduces the oil viscosity. The more the solvent reduces the viscosity, the less this specific time will be. If this time is small for a system, the concentration in the interface of mobile/immobile layers is very low. Figure 4 shows the solvent concentration profile in the diluted oil for different values of specific time,  $t_D^*$ . It should be noted that the value of  $\alpha$  does not change the concentration profile for a fixed value of  $t_D^*$ .

## Stability Analysis

By using stability analysis, we investigate whether convection currents occur or not when CO<sub>2</sub> increases the density of oil by dissolution. In natural convection processes, there is a critical Rayleigh number beyond which convection will occur. The coordinate system of the equations is the one shown in Figure 2. The model is a homogeneous porous medium with constant thickness. The governing equations of the dissolution and oil flow are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (14)$$

$$u = -\frac{k}{\mu} \left( \frac{\partial p}{\partial x} - \rho g \cdot \cos \theta \cdot \nabla z - \rho g \cdot \sin \theta \cdot \nabla x \right) \quad (15)$$

$$D_1 \phi \cdot \nabla^2 C - V \cdot \nabla C = \phi \frac{\partial C}{\partial t} \quad (16)$$

$$\rho = \rho_0 (1 + \gamma \cdot C) \quad (17)$$

In the above equations,  $u$ ,  $v$  and  $w$  are the components of velocity vector,  $\mathbf{V}$ , in the  $x$ ,  $y$  and  $z$  directions, respectively. Because of the choice of the coordinate system, the gravity has two components in the  $x$  and  $z$  directions.  $D_1$  is the molecular diffusivity of CO<sub>2</sub> into mobile oil, and  $\gamma$  is the amount of density increases by the solvent concentration.

Perturbation equations are derived from the above equations with disturbed quantities of velocity,  $u'$ ,  $v'$  and  $w'$ , and concentration,  $C'$ . The perturbed equations are:

$$\nabla^2 w'_D = \frac{kgL \Delta \rho \cdot \cos \theta}{\mu \phi D_1} \left[ \nabla_H^2 C'_D - \frac{\partial^2 C'_D}{\partial x_D \partial z_D} \tan \theta \right] \quad (18)$$

$$\nabla^2 C'_D - w'_D \frac{\partial C_D}{\partial z_D} - U_D \frac{\partial C'_D}{\partial x_D} = \frac{\partial C'_D}{\partial t_D} \quad (19)$$

where  $C'_D$  is the dimensionless perturbed concentration,  $C_D$  is the dimensionless concentration of the solvent in the boundary

layer,  $t_D = \frac{t \cdot D_1}{L^2}$  is the dimensionless time,  $x_D$  and  $z_D$  are

dimensionless coordinates,  $\nabla_H^2 = \frac{\partial^2}{\partial x_D^2} + \frac{\partial^2}{\partial y_D^2}$  is the 2D-

Laplacian operator,  $w'_D = \frac{w' L}{\phi \cdot D_1}$  is the dimensionless perturbed

velocity in the  $z$  direction,  $U_D = \frac{U \cdot L}{\phi \cdot D_1}$  is the dimensionless

flow velocity in the  $x$  direction with  $U$  being the flow velocity in this direction,  $\Delta \rho = \rho_0 C_s$  is the density difference between the CO<sub>2</sub>-saturated oil and the live oil, and  $\theta$  is the angle of the mobile oil layer with the horizontal.

The term,  $Ra = \frac{kgL \Delta \rho \cdot \cos \theta}{\mu \phi D_1}$ , is the Rayleigh number of the

system that determines the possibility of convection. In

Equation (19),  $\frac{\partial C_D}{\partial z_D}$  should be calculated. In the previous

section, concentration as a function of  $z$  was estimated. For derivatives, only the first five terms were used for calculations. Neglecting other terms beyond these ones creates only a very small amount of error, which can safely be eliminated.

Perturbed velocity ( $w'_D$ ) and concentration ( $C'_D$ ) can be expressed as:

$$\begin{bmatrix} w'_D \\ C'_D \end{bmatrix} = \begin{bmatrix} w_D^*(t_D, z_D) \\ C_D^*(t_D, z_D) \end{bmatrix} \exp[i(a_x x_D + a_y y_D)] \quad (20)$$

where  $w_D^*$  and  $C_D^*$  are amplitudes of the perturbed quantities, and  $a_x$  and  $a_y$  are wave numbers in the appropriate directions satisfying  $a = \sqrt{a_x^2 + a_y^2}$ , with  $a$  being wave number in the  $x$ - $y$  plane. Using the Galerkin technique, the amplitudes can be expressed as<sup>[19, 20]</sup>:

$$w_D^* = \sum_{l=1}^N A_l(t_D) \exp\left[i \left(\frac{2l-1}{2} \pi z_D\right)\right] \quad (21)$$

$$C_D^* = \sum_{l=1}^N B_l(t_D) \exp[i(l\pi z_D)] \quad (22)$$

where the real part of  $w_D^*$  and the imaginary part of  $C_D^*$  satisfy boundary conditions.  $w_D^*$  is expressed in this manner, because the free surface boundary condition exists at the solvent chamber/mobile oil interface ( $z_D=0$ ); and, at the interface of mobile oil and immobile oil ( $z_D=1$ ), the no-slip boundary condition exists. Solvent concentrations at both ends of the mobile strip are constant; therefore,  $C_D^*$  is zero at these boundaries. The amplitude functions are written in complex form, because the form of perturbed equations necessitates

using complex functions for amplitudes. The summation is for 1 to  $N$ , where  $N$  is sufficiently large so that the error encountered by neglecting numbers beyond  $N$  is small.

The amplitudes,  $A_l(t_D)$  and  $B_l(t_D)$ , determine the growth or decay of perturbations. By substituting the above equations into the perturbed equations and using the orthogonal property of trigonometric functions, the following equations can be obtained:

$$E_{lm} A_l(t_D) = \sum_{m=1}^N Ra.(a^2 - a_x.l\pi.\tan\theta)F_{lm}B_l(t_D) \dots\dots\dots (23)$$

$$\delta_{lm} \frac{dB_l(t_D)}{dt_D} = -G_{lm}B_l(t_D) - 2 \sum_{m=1}^N I_{lm}A_m(t_D) - \sum_{\substack{m=1 \\ m \neq l}}^N a_x.U_D.J_{lm}B_l(t_D) \dots (24)$$

where:

$$E_{lm} = \delta_{lm} \left[ a^2 + \left( \frac{2l-1}{2} \pi \right)^2 \right] \dots\dots\dots (25)$$

$$F_{lm} = \frac{8m(-1)^{l-m}}{\pi(2m-2l+1)(2l+2m-1)} \dots\dots\dots (26)$$

$$G_{lm} = \delta_{lm} [a^2 + (l\pi)^2] \dots\dots\dots (27)$$

$$I_{lm} = \int_0^1 \sin(m\pi z_D) \sin\left(\frac{2l-1}{2} \pi z_D\right) \left( \frac{\partial C_D}{\partial z_D} \right) dz_D \dots\dots\dots (28)$$

$$J_{lm} = \frac{2l [1 - (-1)^{l+m}]}{\pi (l^2 - m^2)} \dots\dots\dots (29)$$

and  $\delta_{lm}$  is the Kronecker delta function.

Equations (23) and (24) should be solved numerically to give values of  $A_l$  and  $B_l$  as a function of time. The Rung-Kutta fourth-order method was used to solve the equations.

For the initial condition, white noise was used as it gives the fastest growing noise in the stability analysis<sup>[19, 20]</sup>. An amplification factor,  $\bar{c}(t_D)$ , is defined, which is obtained by dividing the disturbances at any time by the initial disturbances<sup>[21]</sup>, so that the growth of the disturbances does not depend on the amplitude of the initial condition of the disturbances.

The initial condition is obtained by setting all the coefficients in Equation (22) to be unity. In Equations (21) and (22), the value of  $N$  is chosen so that sufficient terms are considered and that adding more terms does not vary the results. For high Rayleigh numbers, more terms should be used to be sure that the solution has enough accuracy. For each Rayleigh number, the value of the wave number,  $a$ , which is an unknown, is varied and the value that results in the amplification factor to grow fastest is set as the wave number of that Rayleigh number. The value of  $a_x$  is also unknown. It is varied between zero and  $a$ , and the value that causes the perturbations to grow fastest is selected as  $a_x$ .

One important thing in solving Equations (23) and (24) is the choice of time step. The time step should be chosen so that it is small enough to give stability to the solution and large enough to make a decision on choosing wave numbers based on the growth of the amplification factor. If the time step is too large, it gives false results, and the decision made would be incorrect.

## Results and Discussions

We have analyzed the stability of the diffusive boundary layer in the Vapex process when CO<sub>2</sub> is used as the solvent. Our conclusion of the occurrence of convection currents is based on the magnitude of the amplification factor with time. Figure 5 shows values of the amplification factor as a function of time. Our analysis showed the critical value of the Rayleigh number is 50. If the Rayleigh number of the system is above 50, the amplification factor increases monotonically with time; and, if it is below 50, the amplitude decreases monotonically with time. In similar problems where buoyancy force induces convection currents, usually the amplitude decreases; and, then after some time, it starts to increase exponentially with time. The start of the convection, which is not defined explicitly, can be taken as the time when the amplitude factor becomes unity, which is equal to its initial value. In this case, the situation is different. If the Rayleigh number is greater than the critical Rayleigh number (50), the amplitude factor always increases, meaning that based on our convention, convection currents occur very soon after the beginning of the process. We started the process when the mobile layer had an angle of 60 degrees with the horizontal. Knowing the fact that it takes a long time for the chamber to develop and for the mobile oil strip to have an angle of 60 degrees with the horizontal, we can say safely that convection currents happen when the Rayleigh number is greater than 50. It should be mentioned that the viscosity was considered constant in the mobile oil strip. In real cases, the viscosity varies significantly in the mobile layer. In this case, we can use an average value of the mobile oil layer viscosity for the Rayleigh number.

In the previous section, we mentioned that a moving coordinate system was used. This coordinate system did not have a significant effect on the results of the problem. We concluded that, if convection currents are likely to happen, they occur in a very short time. The reduction in the angle of the mobile layer with the horizontal is very small, and the error encountered is negligible. Therefore, using a moving coordinate system in this problem is acceptable.

In Figure 6, wave number is plotted versus Rayleigh number. As the value of the Rayleigh number increases, the value of the wave number also increases; and, there is a linear relationship between them. These values of wave numbers result in the fastest growth of the amplitude factor. The wave number itself has two components,  $a_x$  and  $a_y$ .  $a_x$  appears in the equations and is unknown. It was found that if  $a_x=0$ , the amplitude factor has the most rapid growth. This means that, in the  $x$  direction (direction of oil flow), there is no pattern repetition. In terms of fluid flow, this indicates that the fluid has a spiral motion in the mobile oil strip towards the production well. This spiral motion causes more mixing in this boundary layer and enhances solvent flux to the mobile oil strip. This spiral motion is repeated in the  $y$  direction with the frequency of  $a_y = a = \frac{2\pi L}{\lambda}$ , where  $\lambda$  is

the wavelength. The value of  $\lambda$  can be found by obtaining  $a_y$  from Figure 6 and using the above equation.

Renner<sup>[22]</sup> measured the diffusion coefficient of CO<sub>2</sub> in oil saturated Berea cores. They reported a higher diffusion coefficient when the cores were vertically oriented; and, the values of the diffusion coefficient in the vertical direction were about 5 times greater than those in the horizontal orientation, which shows the effect of unstable buoyancy-induced perturbations.

The Rayleigh number has different parameters, all of which contribute to the same degree to the occurrence of convection. Permeability,  $k$ , is high for heavy oil and bitumen reservoirs and is in the range of 1~10 darcy, which is high and tends to increase the possibility of convection. The thickness of the boundary layer in the mobile oil strip,  $L$ , is very thin and can be estimated to be a few centimeters. The diffusivity of CO<sub>2</sub> into heavy oils has been investigated by many authors. It is estimated that the value of the diffusivity of CO<sub>2</sub> into heavy oil is about  $1\sim 9 \times 10^{-10} \text{ m}^2/\text{s}$  <sup>[23]</sup> (depending on the viscosity), and it should be divided by tortuosity of the porous medium. Viscosity is a very important parameter in the Rayleigh number. As was previously mentioned, the value of viscosity was considered as an average of the viscosity in the boundary layer; and, because it was still high after dilution of oil, it had to decrease the Rayleigh number and inhibit convection. In experimental scales, the values of all the parameters are in the same order of magnitude with field-scale cases, except for the permeability. The permeabilities in the experiments were much higher than actual field values, due to scaling conditions <sup>[24]</sup>. This high permeability value in the experiments can cause the value of the Rayleigh number to be greater than the critical Rayleigh number of 50 and make convection currents happen. Convection can increase the rate of production because of higher mass transfer across the solvent/oil interface. This phenomenon can explain the unexpected high rate in Dunn's experiment when they used CO<sub>2</sub> as a solvent. In field-scale projects, it is very optimistic to expect the Rayleigh number to be greater than the critical value. The value of the Rayleigh number in field projects seems to be at least an order of magnitude less than the critical Rayleigh number. Li et al. <sup>[17]</sup> reported values of the Rayleigh number for a variety of different parameters. Although their definition of the Rayleigh number was different from our analysis, their data had the Rayleigh numbers in the pilot experiments as higher than the critical Rayleigh number for most of the cases considered.

It is interesting to note that the values of  $\alpha$  and  $R$  did not have significant influences on the results.  $\alpha$  did not change the concentration profile of the solvent; and,  $R$  did not have any contribution to convection, since the amplitude factor behavior does not depend on the value of  $R$ .

## Conclusion

In this study, linear stability analysis was applied to investigate the stability of the diluted oil in the boundary layer in CO<sub>2</sub>-Vapex in a homogeneous formation. The potential of CO<sub>2</sub> to increase the density of heavy oils can contribute to the induction of convection mechanism in Vapex, if CO<sub>2</sub> is used as the solvent. Convection can enhance mass transfer across the solvent chamber/mobile oil interface and increase that production rate beyond what is expected. Our results show a critical value of 50 for the Rayleigh number, above which adverse density gradient causes monotonically increasing amplitude of instabilities. In the experiments done for Vapex, the Rayleigh number can usually exceed this critical value; however, in field-scale problems, the expectation to have Rayleigh numbers greater than the critical value is beyond reality. The difference between the pilot-scale and field applications is the value of the permeability used in the experiments for kinematic similitude. This can explain why, in some experiments, the production rate is more than the expected

rate (Dunn's experiment), but in real-scale projects, this phenomenon is not expected.

## Acknowledgements

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## NOMENCLATURE

$A$ :	Time component of the amplitude function for perturbed velocity
$a$ :	Dimensionless wave number
$B$ :	Time component of the amplitude function for perturbed concentration
$C$ :	Concentration
$D_1$ :	Diffusivity of the solvent in the mobile oil layer
$D_2$ :	Diffusivity of the solvent in the mobile oil layer
$d$ :	Derivative
$E$ :	Square matrix
$F$ :	Square matrix
$G$ :	Square matrix
$g$ :	gravitational acceleration
$H$ :	Formation thickness
$i$ :	Imaginary number, $\sqrt{-1}$
$J$ :	Square matrix
$k$ :	permeability
$L$ :	Thickness of the mobile oil layer
$N$ :	Constant number in the summations
$N_s$ :	Dimensionless parameter
$p$ :	pressure
$q$ :	Flow rate
$R$ :	Flow parameter constant
$r$ :	length of the interface strip
$Ra$ :	Rayleigh number
$S$ :	Area
$t$ :	time
$U$ :	Velocity of the diluted oil in the mobile layer
$u$ :	velocity in the $x$ direction
$V$ :	Volume
$V$ :	Velocity vector
$v$ :	velocity in the $y$ direction
$w$ :	velocity in the $z$ direction
$w_j$ :	width of the
$x$ :	coordinate direction parallel to the flow
$y$ :	coordinate direction
$z$ :	coordinate direction perpendicular to the flow
$\alpha$ :	Constant parameter in the concentration function
$\phi$ :	Porosity
$\gamma$ :	Density coefficient
$\kappa$ :	Square root of the ratio of diffusivities of the solvent into mobile and immobile oil
$\lambda$ :	Wavelength
$\mu$ :	Viscosity
$\rho$ :	Density
$\theta$ :	Mobile oil strip angle with horizontal
$\Delta\rho$ :	Density difference
$\Omega$ :	Cementation factor

$\nabla_H^2$ : 2D Laplacian operator

#### Superscripts

- ' Perturbed state
- \* Amplitude of the perturbations (in  $t_D^*$  denotes constant dimensionless time for concentration)

#### Subscripts

- 0: Base state
- c: Critical
- D: Dimensionless
- l, m: Summation index
- s: Equilibrium state
- x, y: x and y directions

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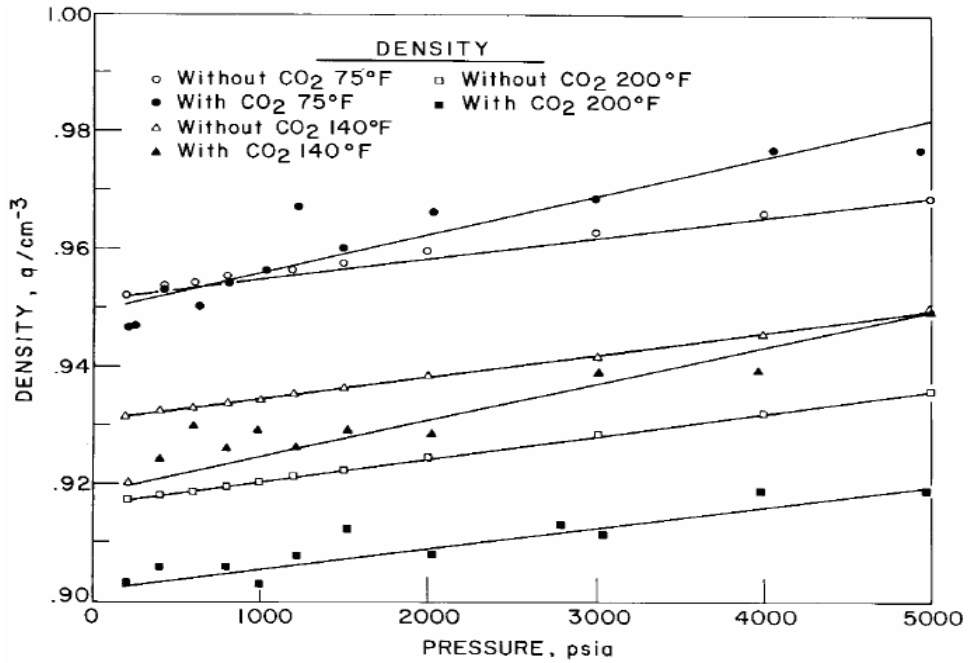


Figure 1. Density of Wilmington oil (17° API Gravity) at three different temperatures [10].

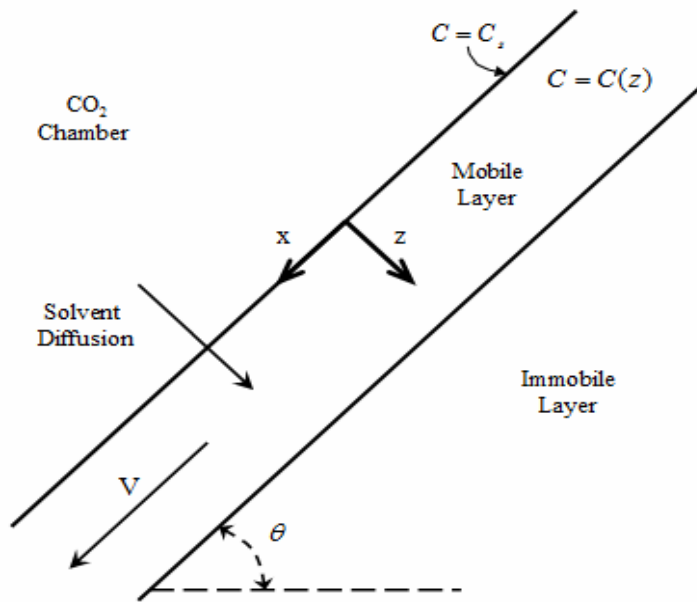


Figure 2. Mass boundary layer in the Vapex process.

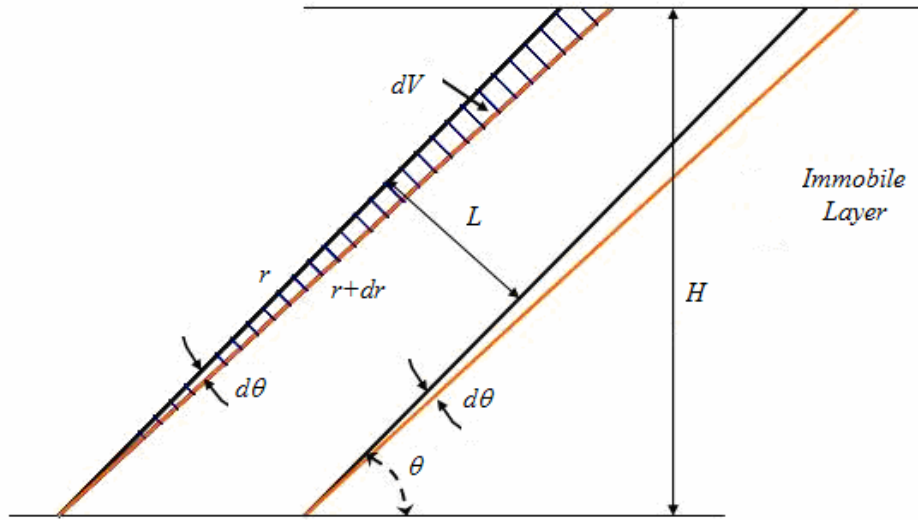


Figure 3. Variation of the angle of mobile layer and its relation to production rate ( $q=dV/dt$ ).

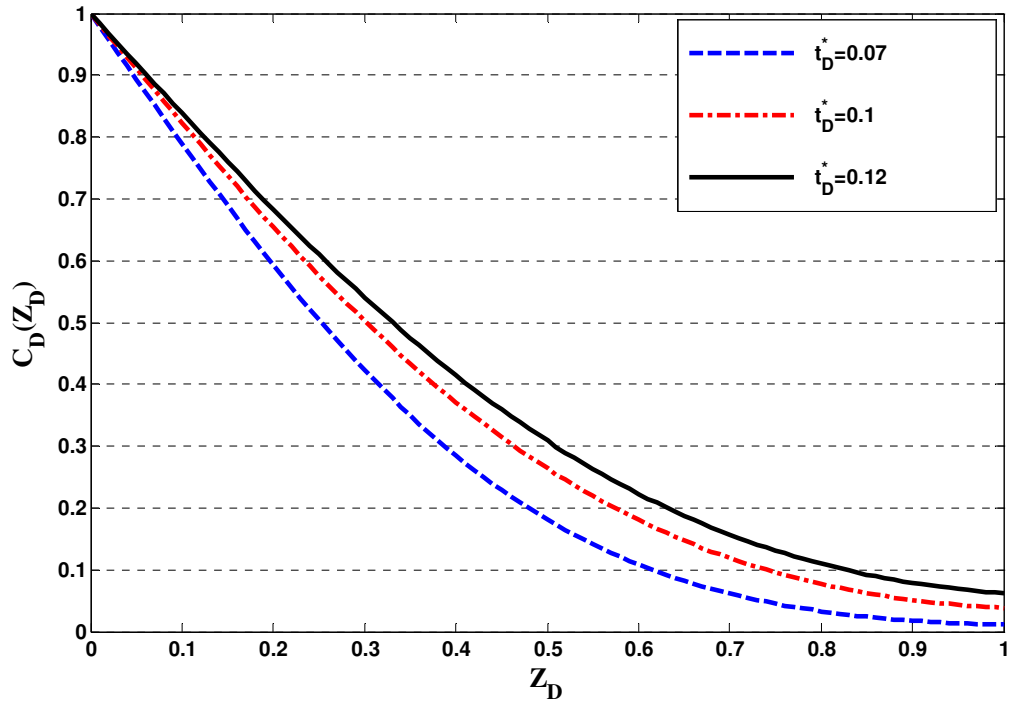


Figure 4. Dimensionless concentration of  $\text{CO}_2$  in the mobile oil layer.

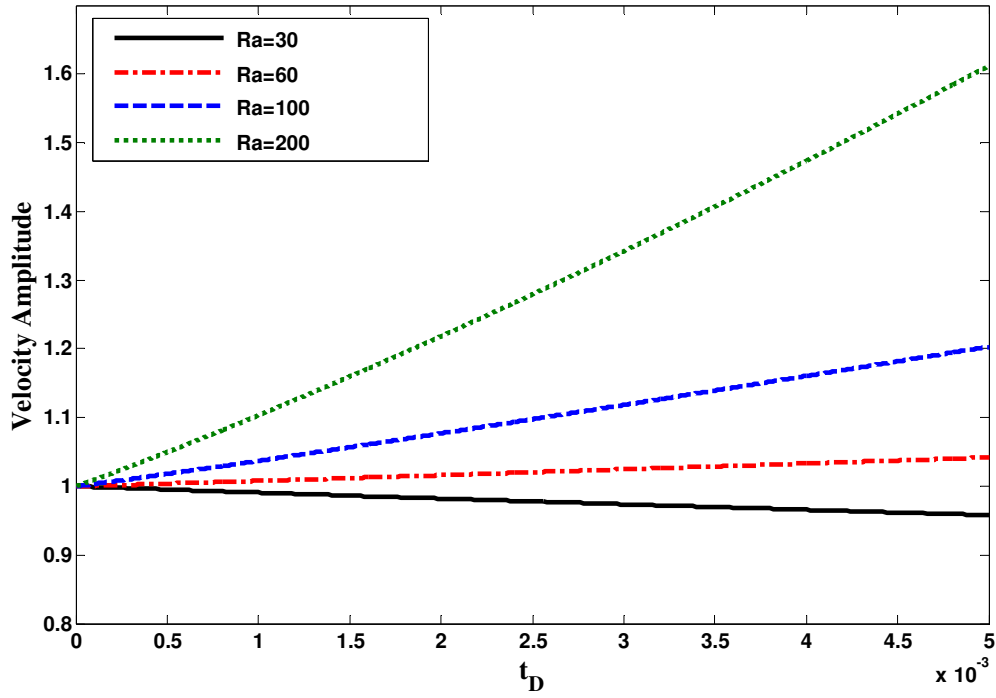


Figure 5. Velocity amplitude growth with time.

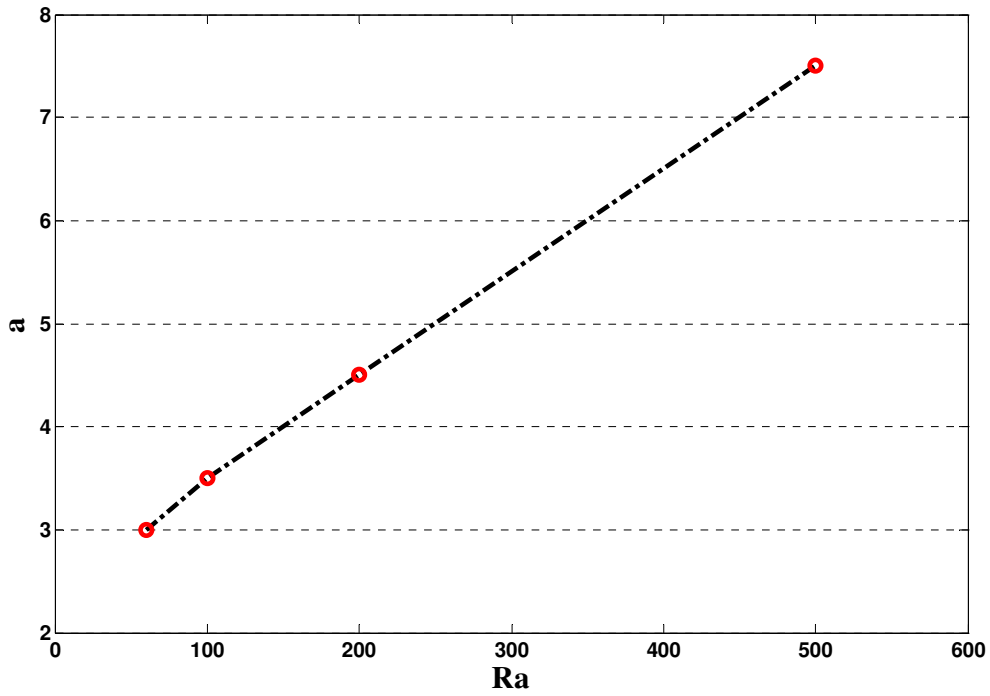


Figure 6. Relationship between wave number and Rayleigh number.